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ABSTRACT

Two approaches for conceptualizing and estimating a contextual model for analyzing student achievement are the separate equation approach and the single equation approach. The separate equation method determines the relationship between the individual-level independent variable and the individual-level dependent variable within each group. It then determines the relationship of the within-group regression intercepts and slopes to the group-level independent variable based on the group mean. In this way, individual, group, and interaction effects are determined. The single equation approach, however, involves a single multiple regression analysis. The group variable and the interaction variable are assigned to each of the individuals in the groups. The partial regression coefficients have the same interpretation in terms of multilevel effects in both methods, but they will not always produce the same estimates of effects. Both approaches to contextual analysis use an ordinary least square estimation procedure, for which common computer packages are available. Computer simulations were carried out on large data sets representing teachers, students, and classes. Results suggest that the separate equation approach provides the most accurate picture of the actual precision of estimates. However, it was more costly. (GDC)

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A COMPARISON OF CONTEXTUAL ANALYSIS MODELS: A SINGLE EQUATION AND A SEPARATE EQUATION TO ANALYZE STUDENT LEARNING OUTCOMES

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A COMPARISON OF CONTEXTUAL ANALYSIS MODELS: A SINGLE EQUATION AND A SEPARATE EQUATION TO ANAYZE STUDENT LEARNING OUTCOMES

It has been accepted that student learning outcomes in a classroom appear to affected by both the student level variables and group level variables. Thus, in much recent educational research, multilevel analyses are recommended for analyzing educational data (Burstein, 1980; Alwin, 1976; Firebaugh, 1979, 1980; Tate, Note 1; Cronbach, 1976; Alexander and Eckland, 1975; Burstein, Fischer and Miller, 1980; Burstein, Linn and Capell, 1978; Burstein and Miller, Note 3).

Multilevel analysis involves at least two or more independent vaiable levels, such as an individual level and an aggregate level, to explain an dependent variable. Contextual analysis is an analytical model which involves both individual and group level variables. Contextual analysis is the study of the effect of an individual-level independent variable on an individual-level dependent variable after controlling for a group-level independent variable and vice versa (Lazarsfeld and Menzel, 1961; Burstein, 1980; Werts and Linn, 1971; Boyd and Iversen, 1979). Contextual analysis is frequently used in research of learning in classroom and in program evaluation. Burstein (1980) explained why most studies of educational effects should involve contextual analyses:

> Schooling activities occur within hierarchical organizations in which the sources of educational influence on students occur in the groups to which an individual belongs. These groups (learning groups within classrooms, classrooms within schools, schools within communities) influence the thoughts, behavior, and feelings of their members. This

> hierarchical structure gives rise to multi-level data. (p. 158)
>
> Attributes of teachers (e.g., instructional practices), classrooms (e.g., ability composition), and schools (e.g., variety of program offerings) are at a macro-level with respect to students, analyses involving group-level explanatory variables and student-level outcomes are quite prevalent in education. (p. 196)

This study focuses on two approaches for conceptualizing and estimating a contextual model. The first is called the SEPARATE EQUATION APPROACH. This approach requires the study of relationships within groups, and requires a two-step procedure. The first step is to determine the relationship between the individual-level independent variable and the individual-level dependent variable within each group. The second step is to determine the relationship of the within-group regression intercepts and slopes (from the first step) to the group-level independent variable based on the group mean. In this approach, individual, group, and interaction effects are determined. The second approach to contextual analysis is called the SINGLE EQUATION APPROACH, and involves a single multiple regression analysis. In this approach, the group variable and the interaction variable are assigned to each of the individuals in the groups. The partial regression coefficients have the same interpretation in terms of multilevel effects as in the separate equation approach. These two approaches will not always produce the same estimates of effects(Boyd and Iversen, 1979).

Conklin (Note 4) referred to the single equation approach as the contextual effects models. He pointed out the limitations of the use of the single equation approach in evaluating educational effects. These limitations are, first, that the estimates in this approach lack statistical efficiency due to failure to recognize intraclass correlations. In addition, this approach assumes that the effects are constant over groups. This second assumption is questionable when one considers the possibility that different classrooms may have different within-group processes. This implies a robustness of within-group dependency which deals with the correlations among the disturbances (Burstein, 1980). Finally, the single equation approach has the error term only at the individual level. This implies that the group-level variable is perfectly specified, which is highly questionable. In contrast, the separate equation approach has error terms both at the group-level and the individual level (Boyd and Iversen, 1979) and allows the within-group regression coeffiecients to vary from group to group.

Since these two approaches are based on different assumptions and computations but are used for the same purpose and both use an ordinary least square estimation procedure, for which programs are available in common computer packages, it was judged important to determine which approach would give the best estimates.

There are also some other multi-level models which are not included in this study, such as the model proposed by Keesling Wiley, a subtraction analysis and a full model. These models have been studied by Kajornsin (1981).

The general purpose of the study was to analyze student learning outcomes using simulated data and two different approaches for estimation. A simulation model uses the similarity concept of "local population" suggested by Cronbach (1976) in which the within-group intercepts are representative of a within-group process, which assumes that the within-group regression coefficients are random across groups. This assumption seems necessary to adequately represent reality (Conklin, Note 4).

The specific purpose of the study was to determine which approach, the single equation approach, or the separate equation gave the best estimates of the effects with the least amount of bias, and whether the apparent precision of the estimates associated with the procedure accurately represented the actual precision.

BASIC MODEL OF THE CONTEXTUAL ANALYSIS

The basic formula for contextual analysis for a single continuous dependent variable, Y, and a single continuous independent variable, X, with a single group-level variable based on group mean, $\bar{X}$, can be written as follows:

$$Y_{ij} = b_o + b_s X_{ij} + b_c \bar{X}_j + b_{sc} X_{ij} \bar{X}_j + w_{ij} \qquad (1)$$

Where:

Y_{ij} and X_{ij} are scores for student i nested within class j. These are the individual-level variables.

$\bar{X}_j$ is the group mean score of X. It is the group-level variable.

$X_{ij} \bar{X}_j$ is the interaction variable.

w_{ij} is the the error term at the individual level. It is assumed that w_{ij} has zero mean and constant variance over levels of X and is uncorrelated with X. The values of w_{ij} are uncorrelated (Firebaugh, 1979).

b_o is an additive constant coefficient of the model.

b_s is a measure of the individual main effect.

b_c is a measure of the group or contextual main effect.

b_{sc} is a measure of the interaction effect.

The equation (1) above specifies that:

> An individual-level dependent variable is a function of a set of explanatory variables consisting of an individual-level variable, a group variable, and an interaction variable. The objective of this model is to detect the presence or absence of effects on the dependent variable associated with each of the three explanatory variables. (Boyd and Iversen, 1979, p. 34)

Most studies using contextual analysis are based on additive models (Firebaugh, 1979, 1980; Alwin, 1976; Alexander and Eckland, 1975). The additive model implies that the interaction effects are not significant. In the original equation (1), if the interaction is significant, the model is referred to as the interactive model. Hence, from equation (1), the addtive model is written as follows:

$$Y_{ij} = b_o + b_s X_{ij} + b_c \bar{X}_j + w_{ij} \qquad (2)$$

Examples of the additive and the interactive models are shown in Figure 1 and Figure 2 (see Appendix A) respectively. Figure 1 shows the relationship between variable X and variable Y for three classrooms with different mean scores of X, $\bar{X}$, (each line represents a classroom). The student effect is represented by the slopes of the regression lines, the different intercepts of the regression lines show the presence of a group effect. For

an additive model, the slopes of the regression lines are parallel, although the intercepts of the regression lines are different. Hence, two students, from different classrooms, having the same score (X) will, on the average, have different Y scores.

Figure 2 is an example of an interactive model of contextual analysis. This figure shows the relationship between variable X and variable Y for three classrooms with different mean scores ($\bar{X}$); the lines represent classrooms. The individual effect is represented by the slope of the regression lines. The different intercepts of the regression lines show the presence of a group effect. The slopes of the regression lines are not constant, showing the presence of an interaction effect.

Erroneous interpretations can occur if the effect of the interaction is significant, but one conducts contextual analysis based on the additive model, without testing whether the effect of interaction is significant. Tate (Note 1) illustrated the different results found between the interactive model and the additive model. The size of the individual effect using the interactive model was ($-1.57 + 0.0295\bar{X}_j$) compared to 0.269 for the additive model, and the size of the group effect using the interactive model was ($-1.54 + 0.0295X_{ij}$) compared to 0.246 for the additive model. This means that when using an additive model the individual effect is constant ($b_s = 0.269$), where in the interactive model, the individual effect varied depending on the value of the group-level variable ($b_c = -1.57 + 0.0295\bar{X}_j$). The coefficient b_{sc} in equation (1) implies that the relationsh p between variable X and variable Y varies as a function of the

group variable $\bar{X}$ (Burstein, 1979).

THE SEPARATE EQUATION APPROACH TO CONTEXTUAL ANALYSIS

The separate equation approach discussed by Boyd and Iversen (1979) is used to detect individual, group, and interactive effects for contextual analysis. This approach requires the study of relationships within groups. Boyd and Iversen believed that within-group variables are functions of group variables (e.g., group mean). These within-group variables are a within-group intercept and within-group slopes. If the separate equation is used and the group variable is based on the group mean, then the within-group relationships can be studied with the following equations (Boyd and Iversen, 1979, p. 37)

$$b_{oj} = b_o + b_c\bar{X}_j + u_j \quad (3)$$

$$b_{sj} = b_s + b_{sc}\bar{X}_j + v_j \quad (4)$$

Where:

b_{oj} and b_{sj} represent a measure of the within-group intercept and within-group slope respectively. The values of b_{oj} and b_{sj} are computed for each group using a linear regression model, namely;

$$Y_{ij} = b_{oj} + b_{sj}X_{ij} + e_{ij} \quad (5)$$

u_j and v_j represent the error terms at the group level. The values of v_j and u_j are said to be the effects on the intercepts and slopes of all group variables other than the group mean. The estimates of b_o, b_s, b_c, and b_{sc} in equation (3) and equation (4) have the same interpretation as the estimates of these effects in equation (1).

THE SINGLE EQUATION APPROACH TO CONTEXTUAL ANALYSIS

The single equation approach for contextual analysis is represented by equation (1) for the interactive model, and equation (2) for the additive model. The single equation approach requires only an Ordinary Least Square analysis to simultaneously estimate the individual, group, and interaction effects. Both the single equation approach and the separate equation approach are used for estimating the effects of b_o, b_s, b_c, and b_{sc}. It is noted that if one substitutes equation (3) and equation (4) into equation (5), then the result would be:

$$Y_{ij} = (b_o + b_c\bar{X}_j + u_j) + (b_s + b_{sc}\bar{X}_j + v_j)X_{ij} + e_{ij} \qquad (6)$$

After rearranging equation (6) and assuming $w_{ij} = v_j X_{ij} + u_j + e_{ij}$ equation (6) becomes similar to equation (1). In the same manner, if the value of b_{sc} in equation (4) is equal to zero, by substituting equation (3) and equation (4) into equation (5), the result will be similar to equation (2). Since these two approaches are used to determine the individual, group, and interaction effects, this gives rise to the question of which of these two approaches gives the best estimate of the effects in terms of accuracy and precision.

SIMULATION PROCEDURE AND METHODOLOGY

A simulation model was used in this study to generate the empirical sampling distributions for an interactive model. The logic, assumptions, and procedures in this simulation model were also used by Tate and Wongbundhit (Note 2).

The Assumptions of the Simulation Model:

The fundamental assumption of this simulation model is that the within-group process appears as a relationship between an individual independent variable, X, and an individual dependent variavble, Y. The individual independent variable can be pretest score, IQ, or any other individual characteristic varible. The individual dependent variable can be a posttest score or an achievement score. A fixed number of students form a class, which assumes that a sirgle teacher teaches a single class. Each student within-class is randomly selected from a student population of the teacher. This population is comparable to a class nested within a local population model for statistical inference (Cronbach, 1976). The within-group process gives the relationship between variable X and variable Y in terms of

$$Y_{ij} = b_{oj} + b_{sj} X_{ij} + w_{ij} \qquad (3.1)$$

Where:

b_{oj} and b_{sj} are the within-group parameters, Y_{ij} and X_{ij} represent corresponding scores for student i within-class j, and w_{ij} represents variables other than X which also have an effect on variable Y.

The within-group parameters, b_{oj} and b_{sj} are assumed to be functions of the group-level variable. In this case, the group-level variable is based on the group mean of the individual independent variable (X). The within-group parameter can be representd by the following mathematical model:

$$b_{oj} = b_o + b_c \bar{X}_j + u_j \qquad (3.2)$$

$$b_{sj} = b_s + b_{sc} \bar{X}_j + v_j \qquad (3.3)$$

Where: b_o, b_s, b_c, and b_{sc} are the structural parameters.

Equations 3.1 to 3.3 represent a model in which the within-group parameters (b_{o_j} and b_{s_j}) are random variables. The assumption of within-group parameters, to be varied randomly over groups, is supported by Burstein, Linn and Capell (1978). Conklin (Note 4) points out that the random coefficient models give efficient parameter estimates which are unbiased and consistent. He also said that it is possible to assume that the educational effects of instruction, peer interaction, school policies, etc., may vary from group to group. Thus, the possibility of heterogenous within-class slopes is a potentially important source of within-group variability. (Burstein, Linn and Capell, 1978). Therefore the simulation for this study is based on the random coefficients model to better represent reality (Tate and Wongbundhit, Note 2).

Simulation Model Parameters:

Four types of parameters are used for generating data for the interactive model populations. These four types of parameters are (1) the study parameters, (2) the structural parameters, (3) the teacher parameters, and (4) the disturbance parameters.

(1) Study parameters are described as follows:

1.1 the number of students in each class (n); it is assumed that the number of students in each class is equal;

1.2 the number of classes for each sample (j).

For this study, a class size of 30 and 10 classes were chosen as representative of moderate size classroom research.

(2) The structural parameters consist of four parameters:

2.1 the additive constant (b_0),

2.2 the individual effect (b_s),

2.3 the group effect (b_c),

2.4 the interaction effect (b_{sc}).

The structural parameters for the interactive model (b_0, b_s, b_c, and b_{sc}) represent the situation in which all the effects are considered to be of practical significance. Figure 2 (see Appendix A) represents the population model for the interactive model. The value of all structural parameters for the model, based on an intraclass correlation (η^2) of 0.4 and an intraclass correlation of 0.2, are shown in Table 1 and 2 (see Appendix B) respectively.

(3) Four teacher parameters were defined:

3.1 the population mean of group mean X, $\mu_{\bar{x}}$,

3.2 the population standard deviation of the group mean X, $\sigma_{\bar{x}}$

3.3 the population mean of the standard deviation of X, μ_s

3.4 the population standard deviation of the standard deviation of X, σ_s .

Values chosen for the teacher parameters for the interactive models are also shown in Tables 1, and 2 (see Appendix B) respectively. The criteria for choosing these values are:

1. The simulation model is based on an arbitrary scale of X This scale has a population mean (μ_x) of 50 and a population variance (σ_x^2) of 100. Assuming equal class sizes, the population mean of the group mean X, $\mu_{\bar{x}}$ will be equal to the population mean of X when class memberships are ignored.

2. The intraclass correlation (η^2) is defined as the proportion of between-class variance, $\sigma^2_{\bar{x}}$, to the total variance of X, σ^2_X, ($\eta^2 = \sigma^2_{\bar{x}}/\sigma^2_X$). Therefore the model parameter, $\sigma^2_{\bar{x}}$, would be equal to the product of the intraclass correlation and the variance of X.

3. The pooled within-class variance, $\sigma^2_{X-\bar{X}}$, is computed as the magnitude of the differences between total variance of X, and a between-class variance, when the variable X and $\bar{X}$ are independent, namely:

$$\sigma^2_{X-\bar{X}} = \sigma^2_X - \sigma^2_{\bar{X}} \qquad (3.5)$$

Where:

$\sigma^2_{\bar{x}}$ is as defined above in section (2). Substituting $\eta^2\sigma^2_x$ for $\sigma^2_{\bar{x}}$ in (3.5) gives $\sigma^2_{x-\bar{x}}$ which is equal to $(1-\eta^2)\sigma^2_x$. If $\sigma_{x-\bar{x}}$ is taken as an appropriate value for the mean of the distribution (i.e., for μ_s), then the model parameter, μ_s, is approximately equal to the value of $\sigma_{x-\bar{x}}$.

4. The model parameter, σ_s, is fixed by assuming that it is equal to 0.1 of the (μ_s) of the distribution.

(4) Three disturbance parameters (the error terms) are defined:

4.1 The variance of the error term at the individual level, σ^2_w. This parameter is specified by assuming that the error variance for an "average" class was four times as large as the explained variance ($b_s\sigma^2_{x-\bar{x}}$).

4.2 The variance of error term at the group level, σ^2_u. This value is associated with the u_j disturbance term in $b_{oj} = b_o + b_c\bar{X}_j + u_j$. Assuming that b_{oj}, $\bar{X}_j$, and u_j are variables and b_o and b_c are constant coefficients, then

$\sigma^2_{b_{o_j}} = b_c \sigma^2_{\bar{x}} + \sigma^2_u$. For this simulation model, the value of σ^2_u is assumed to be twice as large as the value of $b_c \sigma^2_{\bar{x}}$.

4.3 The variance of another error term at the group level, σ^2_v. This value is associated with the v_j disturbance term in $b_{s_j} = b_{sc} \bar{X}_j + v_j$. Assuming that b_{s_j}, $\bar{X}_j$, and v_j are variables and b_s and b_{sc} are constant coefficients, then $\sigma^2_{b_{s_j}} = b_{sc} \sigma^2_{\bar{x}} + \sigma^2_v$. For this simulation model, the value of σ^2_v is assumed to be 0.04 times as large as the value of $b_{sc} \sigma^2_{\bar{x}}$.

Two interactive model populations were used in this study. Table 1 and 2 show all the values of simulation model parameters for the interactive model population based on an intraclass correlation of 0.4, and the interactive model population based on an intraclass correlation of 0.2 respectively.

SIMULATION MODEL PROCEDURE

The procedure for generating the data is described below. Figure 3 (see Appendix A) illustrates the simulation procedure.

Step 1, randomly sample a teacher from a population of teachers; assume that there is a population of students associated with each teacher (j) in the population of teachers. The j^{th} student population is assumed to be specified with the mean, μ_{x_j}, and the standard deviation, σ_{s_j}, of the distribution of X for the student population. Sampling the j^{th} teacher corresponds to sampling μ_{x_j} and σ_{s_j} from the normal distributions $N(\mu_{\bar{x}}, \sigma_{\bar{x}})$ and $N(\mu_s, \sigma_s)$ respectively.

Step 2, sample n students to form a class for the teacher. The sampling of students in Step 2 consists of sampling nX values from $N(\mu_{x_j}, \sigma_{s_j})$.

Step 3, the mean of X for the class, $\bar{X}_j$, is computed.

Step 4, within-class intercepts, b_{o_j}, and within-class slopes, b_{s_j}, are computed by using equations (3.2) and (3.3) for the interactive model. The values of u_j and v_j are samples for the normal distribution $N(0,\sigma_u^2)$, and $N(0,\sigma_v^2)$ respectively.

Step 5, compute the dependent variable (Y_{ij}) for each student in the class, using equation (3.1). The values of w_{ij} are samples from the normal distribution of $N(0,\sigma_w^2)$. The values of b_{o_j}, b_{s_j}, and X_{ij} are taken from Step 4 and Step 2 respectively.

Repeat Step 1-4 for j teachers to form one study replication and repeat Step 1-5 to replications for the required sampling distributions.

ESTIMATION PROCEDURE

One thousand and 500 replications of each sample are generated for an interactive model population based on $\eta^2 =$ 0.4, and 0.2 respectively. Each replication is analyzed by both the single equation approach and the separate equation approach to estimate the coefficient effects (b_o, b_s, b_c, and b_{sc}).

Single Equation Approach:

The procedure of estimating b_o, b_s, b_c, and b_{sc} is equation (1). This approach is an Ordinary Least Squares analysis, using the student as unit of analysis, the group variable, based on group mean, and the interaction variable, based on the product of the individual and group variables.

Separate Equation Approach:

A two-step procedure is used in the separate equation approach. The first step is to determine the values of within-class intercept (b_{o_j}), and within-class slope (b_{s_j}) for each class j, by using equation (3.1). The second step is to determine the values of b_o, b_s, b_c, and b_{sc}. The value of b_s and b_{sc} are determined by regressing b_{s_j} on $\bar{X}_j$, using equation (3.3). Determining the values of b_o and b_c, the value of b_{o_j} regresses on $\bar{X}_j$, using equation (3.2). There are two alternatives for b_{o_j}, used to regress on $\bar{X}_j$ to determine the value of b_o and b_c. One called "actual within-group intercept (actual b_{o_j})" and the other one is called "adjusted within-group intercept (adjusted b_{o_j})". An actual b_{o_j} is the actual value of the within-group intercept, computing directly from equation (3.1). The value of the adjusted b_{o_j} is equal to $(\bar{Y}_j - (b_s + b_{sc}\bar{x}_j)\bar{X}_j)$. Using the value of the adjusted b_{o_j} to regress on $\bar{X}_j$, was suggested by Tate and Wongbundhit (Note 2). Thus, the separate equation approaches using both the actual b_{o_j} were used for comparison in this study.

SAMPLING DISTRIBUTION OF THE ESTIMATES OF THE EFFECTS

After all replications of each sample have been analyzed by alternate approaches, the following values are computed:

1. the estimated expected value, E(b), of the estimate, which is the mean of the empirical sampling distribution of the estimate;

2. the estimated standard error of the estimate, "actual SE(b)"; which is the standard deviation of the sampling distribution of the estimates.

3. the average apparent standard error of the estimates, "apparent SE(b)", which is the mean of the sampling distribution of the apparent standard error of the estimate.

The bias of estimates is the amount of difference between E(b) and the true parameter. The smaller values of bias and "actual SE(b)" for the estimates are the better estimates. The other criterion used to judge which of the alternate approaches gives the best estimates is the magnitude of the difference between the value of the "apparent SE(b)" and the actual SE(b)." The magnitude of the difference should be small. When any parameter estimate has the "actual SE(b)" equal to "apparent SE(b)," the estimate of precision for this parameter estimate will be approximately equal to the actual value, in the long run.

SIMULATION RESULTS

Data Based on the Intraclass Correlation of 0.4:

The results of the analyses of the 1,000 replications for the single equation, the separate equation approach using actual b_{o_j}, and the separate equation approach using adjusted b_{o_j} are shown in Table 3. Results of the statistical tests of bias of the estimates (for b_o, b_s, b_c, and b_{sc}) are included in Table 4.

Results of the statistical tests of bias of the estimates for b_o, b_s, b_c, and b_{sc} indicate that the single equation approach, the separate equation approach using adjusted b_{o_j}, and the separate equation approach using actual b_{o_j} provide unbiased estimates for all four parameters. The "actual SE(b)" for both the separate equation approach using actual b_{o_j} and the separate equation approach using adjusted b_{o_j} are approximately the same.

These "actual SE(b)" values are larger when the single equation is employed.

The associated apparent precision is accurate for both the separate equation approach using actual b_{o_j} and the separate equation approach using adjusted b_{o_j}. The values of "actual SE(b)" are approaximately 1.00 to 1.11 times the values of "apparent SE(b)". For the single equation approach, the values of "actual SE(b)" for b_o, b_s, and b_c are approximately 3.62 to 3.76 times the values of "apparent SE(b)", and the values of "actual SE(b)" for b_{sc} are approximately 14 times the value of "apparent SE(b)". This indicates that the long-range apparent precision of estimation was not accurate for the single equation approach.

Data Based on the Intraclass Correlation of 0.2:

The results of the analyses of the 500 replications for the single equation, the separate equation approach using actual b_{o_j}, and the separate equation approach using adjusted b_{o_j} are shown in Table 5. Results of the statistical tests of bias of the estimates (for b_o, b_s, b_c, and b_{sc}) are included in Table 6.

Results of the statistical tests of bias of the estimates for b_o, b_s, b_c, and b_{sc} indicate that the single equation approach, the separate equation approach using adjusted b_{o_j}, and the separate equation approach using actual b_{o_j} provide unbiased estimates for all four parameters. The "actual SE(b)" for both the separate equation approach using actual b_{o_j} and the separate equation approach using adjusted b_{o_j} are approximately the same. These "actual SE(b)" values are larger when the single equation is employed.

The associated apparent precision is accurate for both the separate equation approach using actual b_{0j} and the separate equation approach using adjusted b_{0j}. The values of "actual SE(b)" are approximately 1.0 to 1.11 times the values of "apparent SE(b)". For the single equation approach, the values of "actual SE(b)" for b_0, b_s, and b_c are approaximately 2.80 to 3.03 times the values of "apparent SE(b)," and the values of "actual SE(b)" for b_{sc} are approximately 17.25 times the value of "apparent SE(b)." This indicates that the long-range apparent precision of estimation was not accurate for the single equation approach.

CONCLUSION

This study proposed to investigate which of the alternate approaches - the single equation approach, the separate equation approach using actual b_{0j}, and the separate equation approach using adjusted b_{0j} - should be selected in order to analyze student learning outcomes in an interactive model population. To answer the above question, data were generated from known parameters by means of a computer simulation model. One thousand replications of the interactive model population based on an intraclass correlation of 0.4 and 500 replications based on an intraclass correlation of 0.2, were generated. Each replication consisted of 10 teachers, with 30 students for each teacher. The major findings were

1. For the interactive model based on an intraclass correlation of 0.4, the results show that both the single equation and separate equation approaches give unbiased estimates

(b_o, b_s, b_c, and b_{sc}). But, the single equation approach produced a larger actual standard error for all estimates than did the separate equation approaches. That is to say, the separate equation approaches gave better actual standard errors for all the estimates than did the single equation approach. Moreover, the results of comparisons between the size of the apparent standard error of the estimates and the size of the actual standard error of the estimates indicated that the separate equation approaches provided an accurate picture of the actual precision of estimates. In contrast, the single equation approach provided an overly optimistic estimate of estimation precision. However, only the separate equation approach using actual b_{o_j} can be considered a better choice than the separate equation approach using adjusted b_{o_j}, as the computations involved in using this approach are less complex than the computations involved in using the separate equation approach using only adjusted b_{o_j}.

2. In order to confirm the conclusions resulting from these analyses, 500 replications of the data were generated based on an intraclass correlation of 0.2. The conclusions of the study from the sample based on an intraclass correlation of 0.2 were the same as the conclusions of the study from the sample based on an intraclass correlation of 0.4. The single equation and the separate equation approaches gave unbiased estimates (b_o, b_s, b_c, and b_{sc}). However, the single equation approach produced large actual standard errors for all estimates than did the separate equation approaches. The results of the comparisons between the size of the apparent standard error of the estimates, and the

size of the actual standard error of the estimates, indicated that the separate equation approaches provided an accurate picture of the actual precision of estimates. In contrast, the single equation appraoch provided an overly optimistic estimate of estimation precision. In other words, the separate equation approach using actual b_{oj} should be selected in order to analyze student learning outcomes in the interactive model. However, this approach is more costly both in the use of computer time and the use of personal time than the use of single equation approach.

Since the results of this study were based on fixed simulation model parameters, they can only generalized to similar parameter values.

RECOMMENDATION FOR FURTHER STUDIES

One possibility for further study would be to generate data for different class sizes, such as 15, 20, 25, and 30 students per class. These data could then be analyzed using the separate equation approach to see which class size will give the best results.

Appendix A

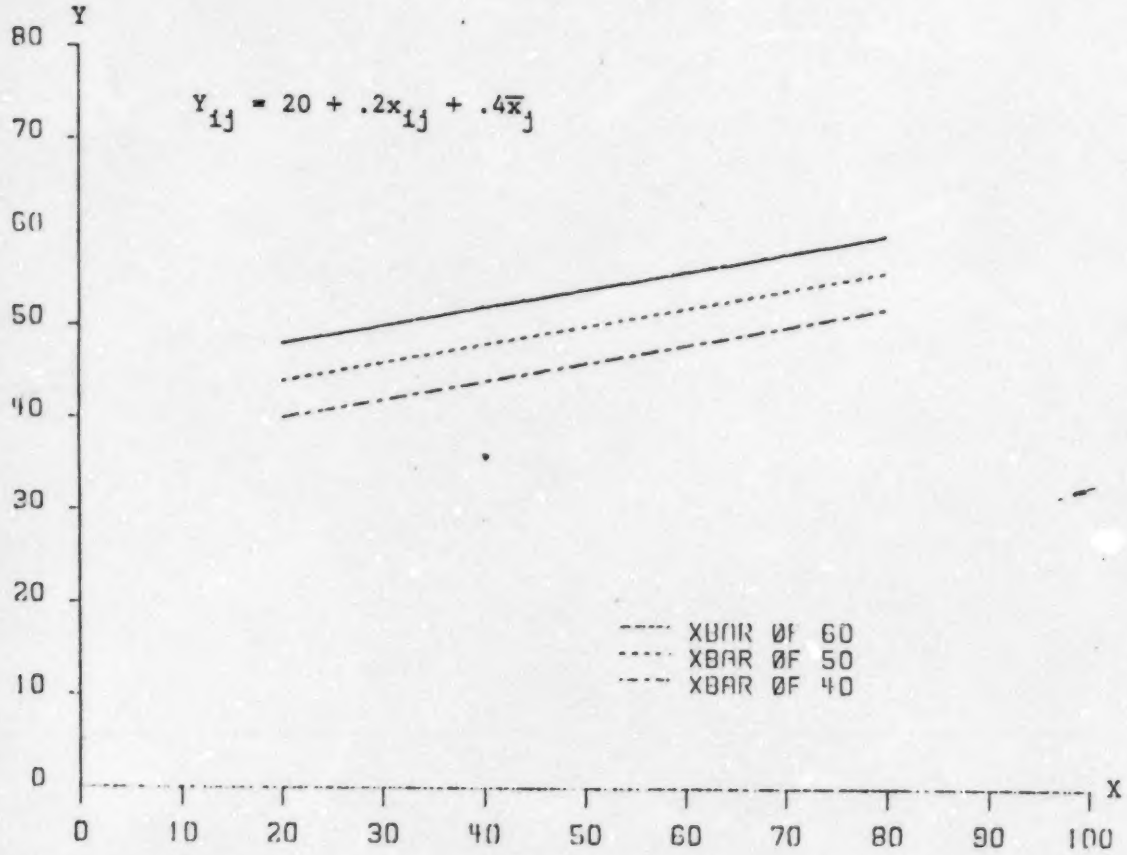

Figure 1. Simulation Model for Additive Model Population

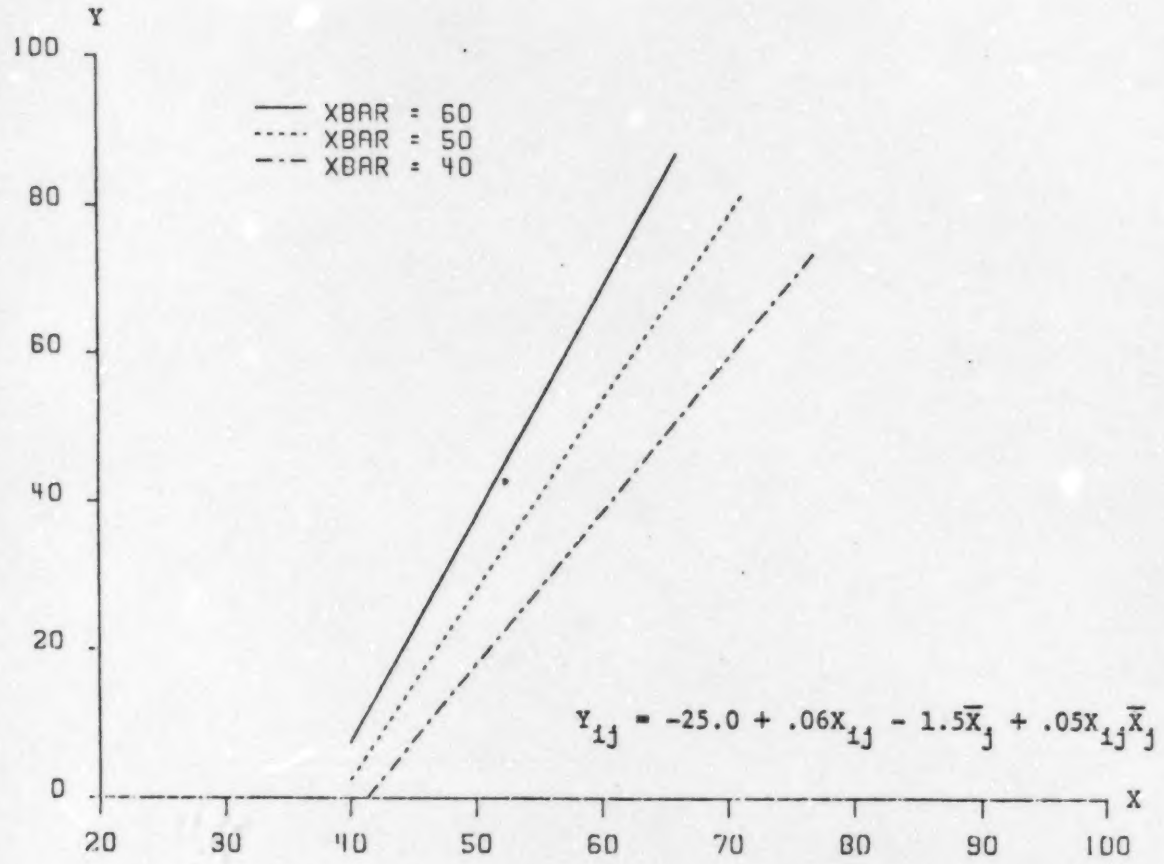

Figure 2. Simulation Model for an Interactive Model Population

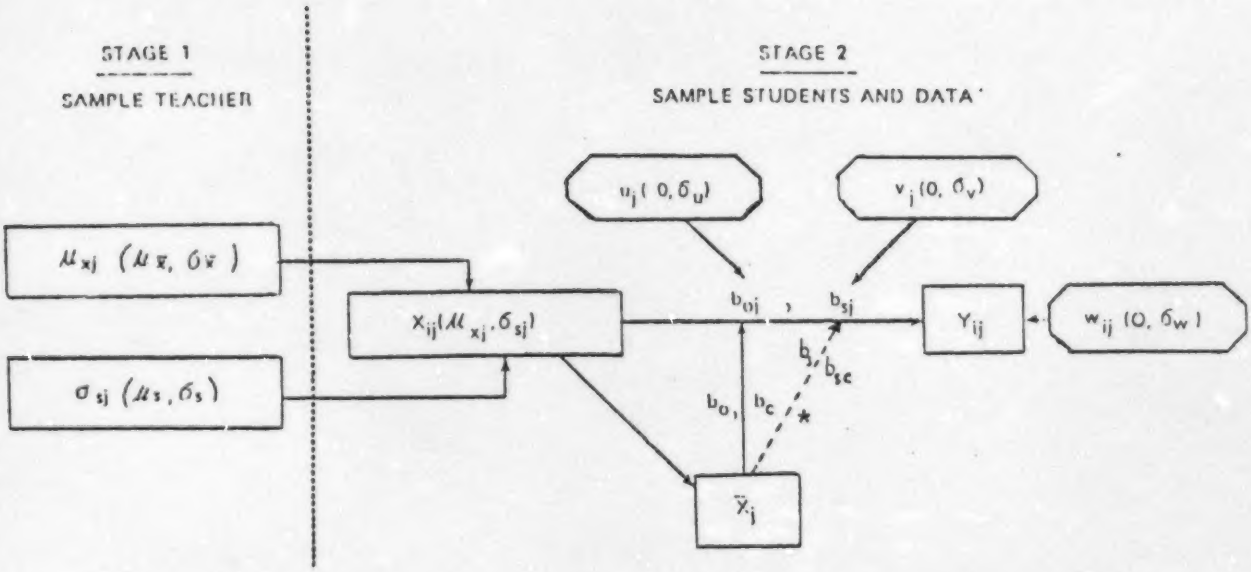

Figure 3. Simulation Procedure

*

The broken line with arrow is included in the simulation procedure for an interactive model. It is not needed in the simulation procedure for an additive model.

Appendix B

Table 1

Simulation Model Parameter for the Interactive Model

Based on the Intraclass Correlation = .4

Study parameters

Student (n) = 30

Teacher (j) = 10

Structural parameters

Additive constant (b_o) = -25.00

Individual effect (b_s) = 0.06

Group effect (b_c) = -1.50

Interaction effect (b_{sc}) = 0.05

Teacher parameters

The population mean of $\bar{x}$ ($\mu_{\bar{x}}$) = 50.0000

The population standard deviation of $\bar{x}$ ($\sigma_{\bar{x}}$) = 6.3245

The population mean of σ_x (μ_s) = 7.7459

The population standard deviation of σ_x (σ_s) = 0.7745

Disturbance parameters

The standard deviation of u (σ_u) = 13.4164

The standard deviation of v (σ_v) = 0.0632

The standard deviation of w (σ_w) = 0.9295

Table 2

Simulation Model Parameter for the Interactive Model

Based on the Intraclass Correlation = .2

Study parameters

Student (n) = 30

Teacher (j) = 10

Structural parameters

Additive constant (b_o) = -25.00

Individual effect (b_s) = 0.06

Group effect (b_c) = -1.50

Interaction effect (b_{sc}) = 0.05

Teacher parameters

The population mean of $\bar{x}$ ($\mu_{\bar{x}}$) = 50.0000

The population standard deviation of $\bar{x}$ ($\sigma_{\bar{x}}$) = 4.4721

The population mean of σ_x (μ_s) = 8.9442

The population standard deviation of σ_x (σ_s) = 0.8944

Disturbance parameters

The standard deviation of u (σ_u) = 9.4868

The standard deviation of v (σ_v) = 0.0447

The standard deviation of w (σ_w) = 1.0733

Table 3

Parameter Estimate Comparison of the

Interactive Model ($\eta^2 = .4$) for 1,000 Replications

Parameter	True Value	E(b)	Actual SE(b)	Apparent SE(b)
		Single Equation		
b_o	-25.00	-25.168	118.640	31.567
b_s	0.06	0.032	2.297	0.633
b_c	-1.50	-1.490	2.383	0.642
b_{sc}	0.05	0.044	0.210	0.015
		Separate Equations (Actual b_{oj})		
b_o	-25.00	-26.056	41.589	37.264
b_s	0.06	0.057	0.205	0.188
b_c	-1.50	-1.478	0.826	0.740
b_{sc}	0.05	0.050	0.004	0.004
		Separate Equations (Adjusted b_{oj})		
b_o	-25.00	-26.058	41.584	38.075
b_s	0.06	0.057	0.205	0.188
b_c	-1.50	-1.478	0.825	0.756
b_{sc}	0.05	0.050	0.004	0.004

Table 4

The Statistical Test of Bias of the

Estimates for the Interactive Model ($\eta^2 = .4$)

Parameter	$\mu(b)$	$\bar{x}(b)$	SD(b)	n	t-value
		Single Equation			
b_o	-25.00	-25.168	118.640	1000	0.0448
b_s	0.06	0.032	2.297	1000	0.3836
b_c	-1.50	-1.490	2.383	1000	-0.1333
b_{sc}	0.05	0.044	0.210	1000	0.8571
		Separate Equations (Actual b_{oj})			
b_o	-25.00	-26.056	41.589	1000	0.8030
b_s	0.06	0.057	0.205	1000	0.5000
b_c	-1.50	-1.478	0.826	1000	-0.1673
b_{sc}	0.05	0.050	0.004	1000	0.0000
		Separate Equations (Adjusted b_{oj})			
b_o	-25.00	-26.058	41.584	1000	0.8046
b_s	0.06	0.057	0.205	1000	0.5000
b_c	-1.50	-1.478	0.825	1000	-0.8462
b_{sc}	0.05	0.050	0.004	1000	0.0000

Table 5

Parameter Estimate Comparison of the

Interactive Model ($\eta^2 = .2$) for 500 Replications

Parameter	True Value	E(b)	Actual SE(b)	Apparent SE(b)
		Single Equation		
b_o	-25.00	-23.926	91.285	30.148
b_s	0.06	0.018	1.668	0.596
b_c	-1.50	-1.505	1.820	0.605
b_{sc}	0.05	0.036	0.276	0.016
		Separate Equations (Actual b_{oj})		
b_o	-25.00	-25.650	37.903	34.927
b_s	0.06	0.051	0.209	0.189
b_c	-1.50	-1.485	0.759	0.696
b_{sc}	0.05	0.050	0.004	0.004
		Separate Equations (Adjusted b_{oj})		
b_o	-25.00	-25.698	37.878	35.831
b_s	0.06	0.051	0.209	0.189
b_c	-1.50	-1.484	0.759	0.714
b_{sc}	0.05	0.050	0.004	0.004

Table 6

The Statistical Test of Bias of the

Estimates for the Interactive Model ($\eta^2 = .2$)

Parameter	$\mu(b)$	$\bar{x}(b)$	SD(b)	n	t-value
		Single Equation			
b_o	-25.00	-23.926	91.285	500	-0.2630814
b_s	0.06	0.018	1.668	500	0.5600000
b_c	-1.50	-1.505	1.820	500	0.0002236
b_{sc}	0.05	0.036	0.276	500	1.1342369
		Separate Equations (Actual b_{oj})			
b_o	-25.00	-25.650	37.903	500	0.3834580
b_s	0.06	0.051	0.209	500	0.9628757
b_c	-1.50	-1.485	0.759	500	-0.4418978
b_{sc}	0.05	0.050	0.001	500	0.0000000
		Separate Equations (Adjusted b_{oj})			
b_o	-25.00	-25.698	37.878	500	0.4120400
b_s	0.06	0.051	0.209	500	0.9628757
b_c	-1.50	-1.484	0.759	500	-0.4713576
b_{sc}	0.05	0.050	0.004	500	0.0000000

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